Mathematical Notation and Some Terminology

- The *operator* L from space W to space E: $L : W \to E$.
- The *kernel* of a linear operator $L : W \to E$ is a subspace $\ker L \subset W$ that transforms by L into 0: $\ker L = \{x \in W | Lx = 0\}$.
- The *image* of a linear operator $L : W \to E$ is a subspace $\mathrm{im} L = L(W) \subset E$.
- *Projector* is a linear operator $P : E \to E$ with the property $P^2 = P$. Projector P is *orthogonal* one, if $\ker P \perp \mathrm{im} P$ (the kernel of P is orthogonal to the image of P).
- If $F : U \to V$ is a map of domains in normed spaces ($U \subset W, V \subset E$) then the *differential* of F at a point x is a linear operator $D_x F : W \to E$ with the property: $\|F(x + \delta x) - F(x) - (D_x F)(\delta x)\| = o(\|\delta x\|)$. This operator (if it exists) is the best linear approximation of the map $F(x+\delta x) - F(x)$.
- The differential of the function $f(x)$ is the linear functional $D_x f$. The *gradient* of the function $f(x)$ can be defined, if there is a given scalar product $\langle | \rangle$, and if there exists a Riesz representation for functional $D_x f$: $(D_x f)(a) = \langle \mathrm{grad}_x f | a \rangle$. The gradient $\mathrm{grad}_x f$ is a vector.
- The *second differential* of a map $F : U \to V$ is a bilinear operator $D_x^2 F : W \times W \to E$ which can be defined by Taylor formula: $F(x + \delta x) = F(x) + (D_x F)(\delta x) + \frac{1}{2}(D_x^2 F)(\delta x, \delta x) + o(\|\delta x\|^2)$.
- The differentiable map of domains in normed spaces $F : U \to V$ is an *immersion*, if for any $x \in U$ the operator $D_x F$ is injective: $\ker D_x F = \{0\}$. In this case the image of F (i.e. $F(U)$) is called the *immersed manifold*, and the image of $D_x F$ is called the *tangent space* to the immersed manifold $F(U)$. We use the notation T_x for this tangent space: $\mathrm{im} D_x F = T_x$.
- The subset U of the vector space E is *convex*, if for every two points $x_1, x_2 \in U$ it contains the segment between x_1 and x_2: $\lambda x_1 + (1-\lambda) x_2 \in U$ for every $\lambda \in [0, 1]$.
- The function f, defined on the convex set $U \subset E$, is *convex*, if its *epigraph*, i.e. the set of pairs $\mathrm{Epi} f = \{(x, g) | x \in U, g \geq f(x)\}$, is the convex set in $E \times R$. The twice differentiable function f is convex if and only if the quadratic form $(D_x^2 f)(\delta x, \delta x)$ is nonnegative.
- The convex function f is called *strictly convex* if in the domain of definition there is no line segment on which it is constant and finite ($f(x) = const \neq \infty$). The sufficient condition for the twice differentiable function f to be

strictly convex is that the quadratic form $(D_x^2 f)(\delta x, \delta x)$ is positive defined (i.e. it is positive for all $\delta x \neq 0$).

- We use summation convention for vectors and tensors, $c_i g_i = \sum_i c_i g_i$, when it cannot cause a confusion, in more complicated cases we use the sign $\sum$.